Graphs of Trigonometric Functions Problems

All answers and graphs must be in radians. Please show a full solution for each problem.

For help with questions 1-3, refer to Examples 1 – 4 on p. 254 – 256.

1. A sine function has an amplitude of 5 and a period of 4π .
   a. Write an equation of the function in the form $y = a\sin kx$
   b. Graph the function over two cycles.

2. A cosine function has a maximum value of 1 and a minimum value of -5.
   a. Determine the amplitude of the function.
   b. Determine the vertical translation.
   c. Write and equation in the form $y = a\cos x + c$.
   d. Graph the function over two cycles.

3. When seated, an average adult breaths in and out every 4 s. The average minimum amount of air in the lungs is 0.08 L, and the average maximum amount of air in the lungs is 0.82 L. Suppose the lungs have a minimum amount of air at t = 0, where t is the time, in seconds
   a. Write a function that models the amount of air in the lungs.
      i. Determine the amplitude, *a*.
      ii. Determine the period and a value for *k*. (i.e Period = $\frac{2\pi}{k}$)
      iii. Determine the vertical shift, *d*, of the function.
      iv. Your function should be in the form $y = a\sin(kx) + d$ or $y = a\cos(kx) + d$.
   b. Graph the function you found in part a.
   c. Determine the amount of air in the lungs at 5.5 s.

For help with questions 4 - 5, please refer to Examples 1 – 4 on p. 263 – 266.

4. Use graphing technology to determine all values of x (in radians) in the interval $[0, 2\pi]$ such that $\sec x = 5$. Round your answers to two decimal places. Draw a sketch of the graph.

5. Describe the graph of $y = \csc x$ in terms of a transformation of the graph of $y = \sec x$. Is there more than one transformation that will accomplish this? Explain your answer.

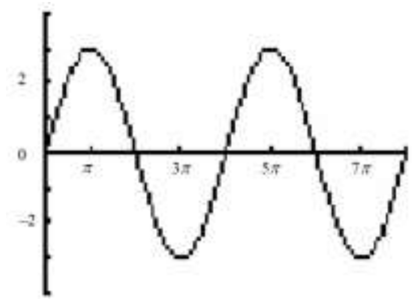
6. a) Explain the difference between $\sec\frac{1}{\sqrt{2}}$ and $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

   b) Determine a value for each expression in part a)

Transformations of Trigonometric Functions Assignment

PART A (Problems 1 – 4) – Provide a full solution for <u>ONE</u> of the following:

1. <u>Describe</u> how you would find the amplitude, period, domain and range of the function $y = 2\cos 4x - 1$.
2. <u>Describe</u> how you would sketch one cycle of the graph $y = \frac{1}{2}\sin 3x$ starting at (0,0).
3. <u>Describe</u> how you would sketch one cycle of the graph $y = 5\cos\frac{1}{2}x$, $-\pi \le x \le \pi$.
4. <u>Describe</u> how to find the amplitude, period, and equation of the graph shown at right.



PART B (Problems 5 – 6)

5. Determine the vertical translation and the phase shift of each function with respect to $y = \cos x$.
   a. $y = \cos x + 6$
   b. $y = \cos\left(x - \frac{\pi}{6}\right) + 2.5$

6. Sketch one cycle of the graph of **ANY 2** of the following. State the amplitude, period, domain, and range of the cycle.

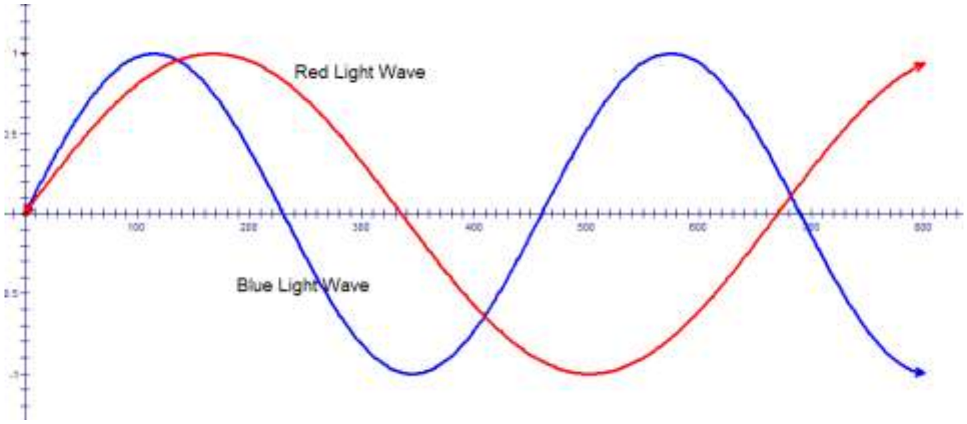
$y = 3\sin x + 2$ $\qquad$ $y = \frac{1}{2}\cos x + 1$ $\qquad$ $y = 3\cos\left(x + \frac{\pi}{4}\right)$

$y = -\frac{1}{2}\sin\left(x + \frac{\pi}{4}\right)$ $\qquad$ $y = 2\cos(x - \pi) - 3$ $\qquad$ $y = \sin 2\left(x - \frac{\pi}{3}\right)$

PART C (Problems 7 – 10) – Provide full solutions for <u>ANY 2</u> of the following:

7. The depth of water, d(t) in metres, in a seaport can be approximated by the sine function $y = 2.5\sin 0.164\pi(t - 1.5) + 13.4$, where t is time in hours.
   a. Graph the function for $0 \le t \le 24$ using a graphing calculator.
   b. Find the period, to the nearest tenth of an hour.
   c. A cruise ship needs a depth of at least 12 m of water to dock safely. For how many hours in each period can the ship dock safely? Round your answer to the nearest tenth of an hour.

8. The sine graphs model waves of red light and blue light, where the units of x are nanometres. Write equations for red lights waves and blue light waves.
   a. Using exact values
   b. Using approximate values to the nearest thousandth



9. The temperature was recorded at 4-h intervals on one summer day. (See Table A)
   a. Graph the sinusoidal curve of best fit and find its equation. Round decimal values to the nearest hundredth.
   b. Use the graph to estimate the temperature at 06:30, to the nearest tenth of a degree.
   c. For what length of time was the temperature at least 25ºC, to the nearest tenth of an hour?
   d. How fast is the temperature changing between 08:00 and 12:00? At 18:00?

- Use STAT EDIT menu to enter the data into two lists.
- Draw the scatter plot using the STAT PLOTS menu.
- Choose suitable WINDOW variables.
- Find the equation of the curve of best fit using SINREG L1, L2, Y1.

10. The table shows the numbers of hours of daylight per day on different days of the year in Thunder Bay. (See Table B)
   a. Graph the sinusoidal curve of best fit and find its equation. Round decimal values to the nearest thousandth.
   b. Find the percent of days in the year with less that 10 h of daylight. Round to the nearest percent.

Time	**Temperature**
00:00	18.1
04:00	15.6
08:00	20.5
12:00	25.4
16:00	28.1
20:00	24.7

TABLE A

Day of Year	**Daylight Hours**
16	8.72
75	11.82
136	15.18
197	15.83
259	12.68
320	9.18

TABLE B

3.8.1 Rate of Change for Trigonometric Functions

Given the function: $f(\theta)=3\sin\left(\theta-\frac{\pi}{6}\right)$

1. Sketch $f(\theta)$ on an interval $\left[\frac{\pi}{6},\frac{7\pi}{6}\right]$

2. Is the function increasing or decreasing on the interval $\frac{\pi}{3}$ to $\frac{2\pi}{3}$?

3. Draw the line through the points $f\left(\frac{\pi}{3}\right)$ and $f\left(\frac{2\pi}{3}\right)$

4. Find the average rate of change of the function $f(\theta)=3\sin\left(\theta-\frac{\pi}{6}\right)$ from $\frac{\pi}{3}$ to $\frac{2\pi}{3}$

   What does this mean?

5. Describe how to find the instantaneous rate of change of $f(\theta)=3\sin\left(\theta-\frac{\pi}{6}\right)$ at $\frac{\pi}{3}$. What does this mean?

3.8.2 Rate of Change for Trigonometric Functions: Problems

For **THREE** of the following functions, sketch the graph on the indicated interval. Find the average rate of change using the identified points, then find the instantaneous rate of change at the indicated point.

1. In a simple arc for an alternating current circuit, the current at any instant ***t*** is given by the function $f(t)=15\sin(60t)$. Graph the function on the interval $0 \le t \le 5$. Find the average rate of change as ***t*** goes from 2 to 3. Find the instantaneous rate of change at ***t*** = 2.

2. The weight at the end of a spring is observed to be undergoing simple harmonic motion which can be modeled by the function $D(t)=12\sin(60\pi\ t)$. Graph the function on the interval $0 \le t \le 1$. Find the average rate of change as ***t*** goes from 0.05 to 0.40. Find the instantaneous rate of change at ***t*** = 0.40.

3. In a predator-prey system, the number of predators and the number of prey tend to vary in a periodic manner. In a certain region with cats as predators and mice as prey, the mice population ***M*** varied according to the equation $M=110250\sin(1/2)\pi\ t$, where ***t*** is the time in years since January 1996. Graph the function on the interval $0\le t \le 2$. Find the average rate of change as ***t*** goes from 0.75 to 0.85. Find the instantaneous rate of change at ***t*** = 0.85.

4. A Ferris Wheel with a diameter of 50 ft rotates every 30 seconds. The vertical position of a person on the Ferris Wheel, above and below an imaginary horizontal plane through the center of the wheel can be modeled by the equation $h(t)=25\sin 12t$. Graph the function on the interval $15 \le t \le 30$. Find the average rate of change as ***t*** goes from 24 to 24.5. Find the instantaneous rate of change at ***t*** = 24.

5. The depth of water at the end of a pier in Vacation Village varies with the tides throughout the day and can be modeled by the equation $D=1.5\cos[0.575(t-3.5)]+3.8$. Graph the function on the interval $0 \le t \le 10$. Find the average rate of change as ***t*** goes from 4.0 to 6.5. Find the instantaneous rate of change at $t=6.5$.