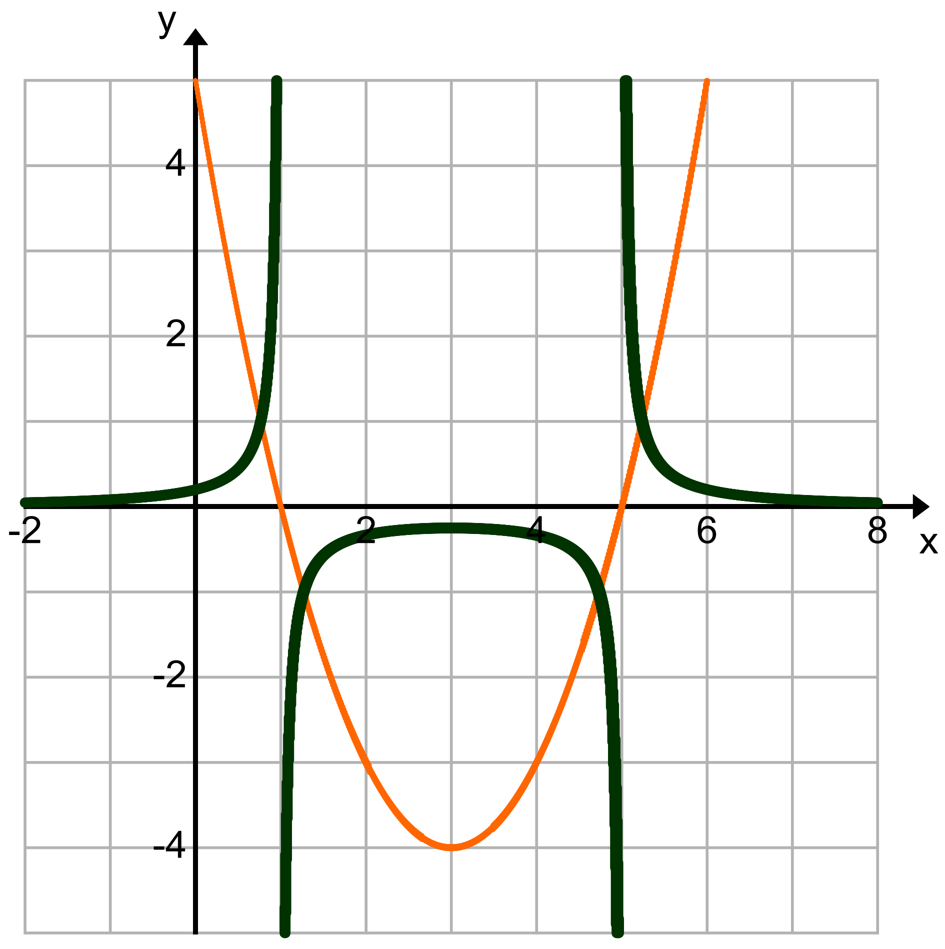
Reciprocals of Quadratic Functions

- The x-intercepts of a quadratic become the vertical asymptotes of the reciprocal.



Rational Functions – Quadratic Reciprocals

- Investigate the properties of the function

$$f(x) = \frac{1}{2x^2 - 9x - 5}$$

  - Intercepts
  - Asymptotes
  - Positive/Negative intervals
  - Domain/Range
  - End Behaviour
  - Rate of Change in each interval

x-intercepts

- For a rational function, only the numerator determines x-intercepts.
- Let f(x) = 0 and solve for *x*.

$$\frac{1}{2x^2-9x-5}=0$$

$$1=0(2x^2-9x-5)$$

$$1\neq 0$$

- $\therefore$ There are no x-intercepts.

y-intercept

- The y-intercept is determined by setting x=0 .
- Then, $f(0) = \frac{1}{2(0)^2 - 9(0) - 5} = -\frac{1}{5}$

- $\therefore$ the y-intercept is $-\frac{1}{5}$

Vertical Asymptotes

- Let Denominator = 0 and solve for *x*
- Factor the denominator

$$(2x+1)(x-5)=0$$

$$x=-\frac{1}{2}, x=5$$

- $\therefore$ the graph's vertical asymptotes are $x=-\frac{1}{2}, 5$

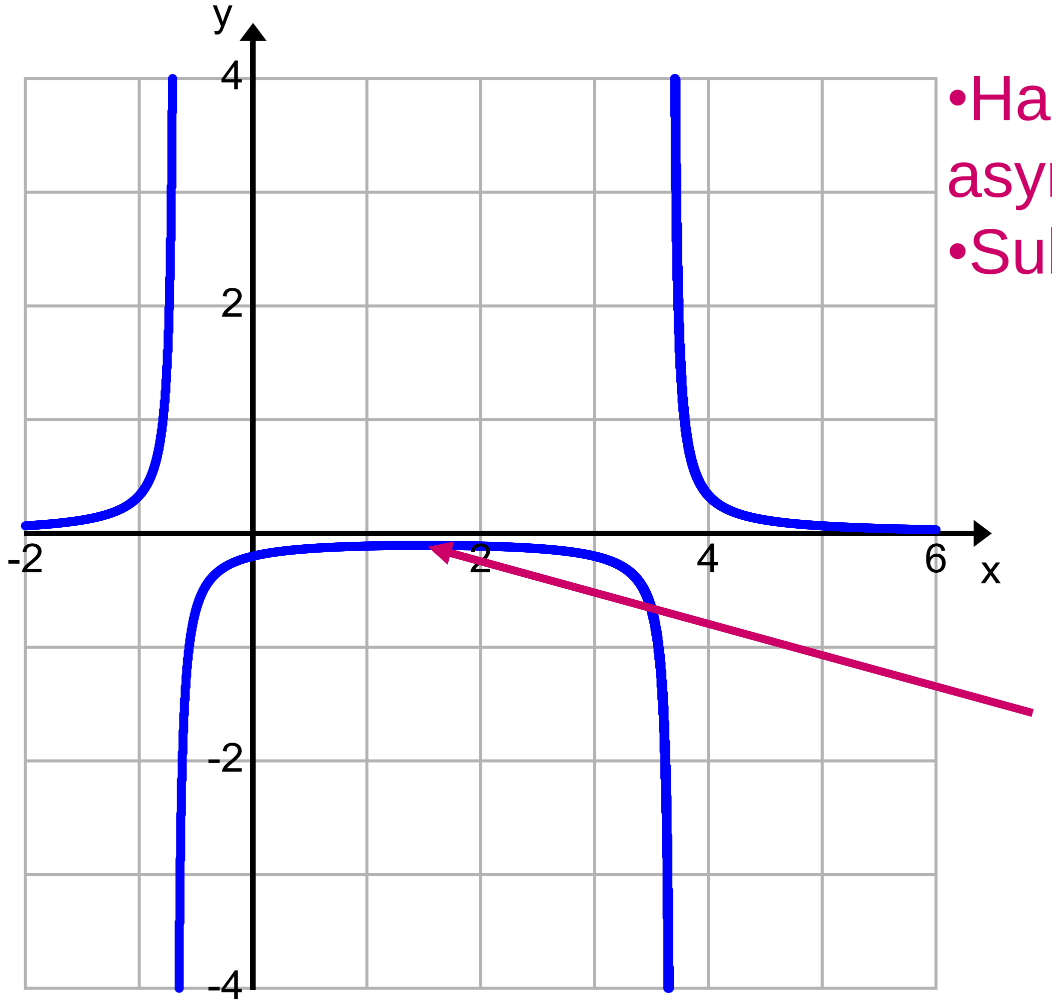
Horizontal Asymptotes

- Examine the behaviour of the function as x → ∞
- Divide all the terms in the function by x^2 (the term with the greatest degree)

$$y = \frac{\frac{1}{x^2}}{\frac{2x^2}{x^2} - \frac{9x}{x^2} - \frac{5}{x^2}} = \frac{\frac{1}{x^2}}{2 - \frac{9}{x} - \frac{5}{x^2}} \to 0 \text{ as } x \to \pm\infty$$

•If the degree of the numerator is less than the degree of the denominator then the horizontal asymptote is the x-axis , y = 0.

Local Max



- Half way between asymptotes
- Substitute into function

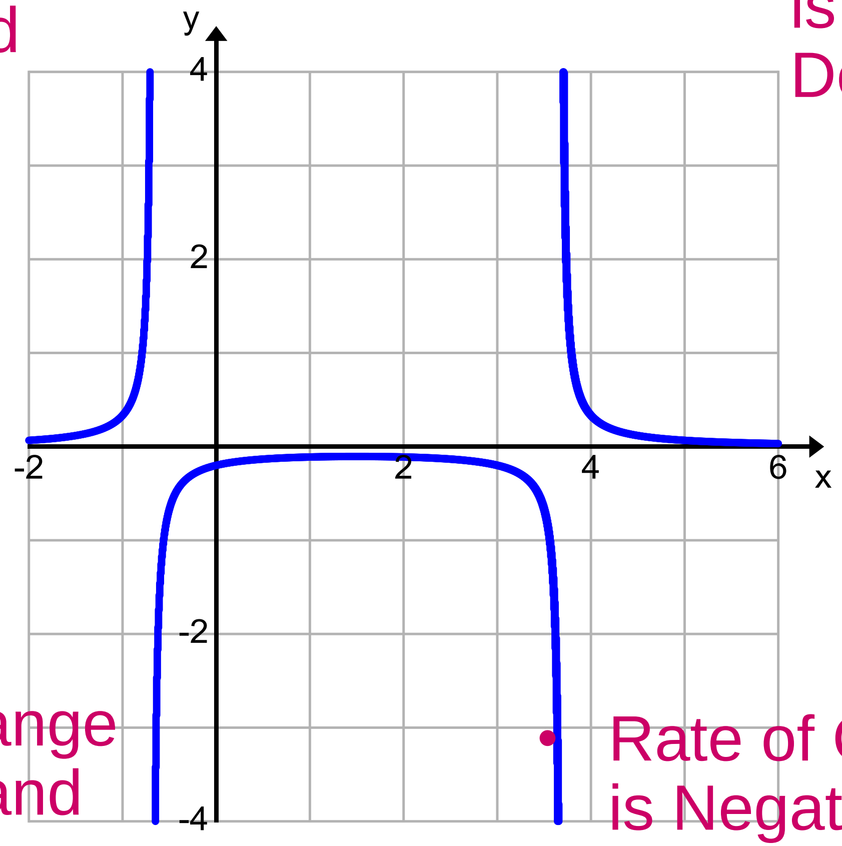
$(2.25, -0.0661)$

Domain/Range/End Behaviour

- Domain $x \in \Re, x \neq -\frac{1}{2}, 5$
- Range $y \in \Re, y > 0, y < -0.0661$
- End Behaviour $f(x) \to 0$

Rate of Change

- Rate of Change is Positive and Increasing
- Rate of Change is Negative and Decreasing



- Rate of Change is Positive and decreasing
- Rate of Change is Negative and Increasing

Rate of Change Summary

Interval	$x < -\frac{1}{2}$	$-\frac{1}{2} < x < 2\frac{1}{4}$	$x = 2.25$	$2\frac{1}{4} < x < 5$	$x > 5$
Sign of f(x)	+	-	-	-	+
Sign of Slope	+	+	0	-	-
Change in Slope	+	-		+	-

The graph is

- positive and increasing with an increasing rate of change for $x < -\frac{1}{2}$
- negative and increasing with a decreasing rate of change for $-\frac{1}{2} < x < 2\frac{1}{4}$
- negative and decreasing with an increasing rate of change for $2\frac{1}{4} < x < 5$
- positive and decreasing with a decreasing rate of change for $x > 5$
- The graph reaches a local maximum on the interval $-\frac{1}{2} < x < 5$ at $x = 2.25$.