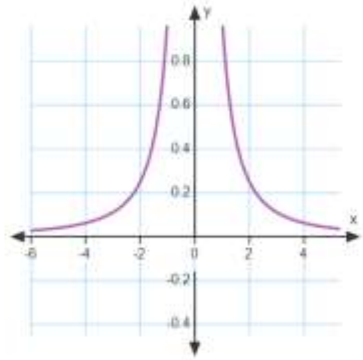
Investigation of the Properties of a Quadratic Reciprocal

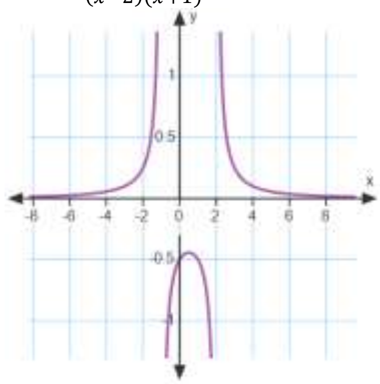
Advanced Functions p. 157

1. $y = \frac{1}{x^2}$



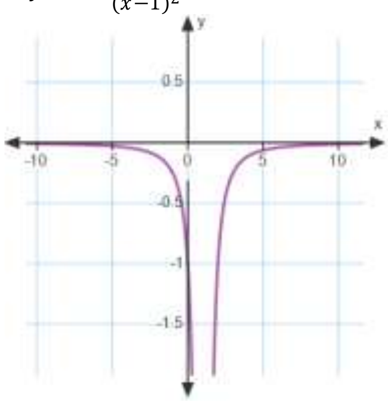
a) Horizontal Asymptote: y = 0 (x-axis)
b) Vertical Asymptote: x = 0 (y-axis)
c) Intercepts: None
d) Domain: $x \neq 0, x \in R$ Range: $y > 0, y \in R$
e) End Behaviour: As $x \to 0^-, f(x) \to \infty$
As $x \to 0^+, f(x) \to \infty$
As $x \to -\infty, f(x) \to 0$
As $x \to +\infty, f(x) \to 0$
f) Positive Intervals: x < 0, x > 0 Negative Intervals: None
g) Increasing Intervals: x < 0 Decreasing Intervals: x > 0

2. $y = \frac{1}{(x-2)(x+1)}$



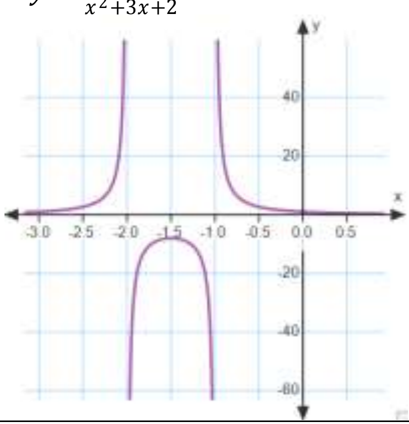
a) Restrictions: $x \neq 2, x \neq -1$ Domain: $x \in R, x \neq 2, x \neq -1$
c) Behaviour of Asymptotes: As $x \to -1^-, g(x) \to \infty$
As $x \to -1^+, g(x) \to -\infty$
As $x \to 2^-, g(x) \to -\infty$
As $x \to 2^+, g(x) \to \infty$
d) Rate of Change: x < -1 Positive Increasing Slope
x > -2 Negative Decreasing Slope
e) Maximum Point is ½ way between the vertical asymptotes and sub into g(x)
f) Max Point $\left(\frac{1}{2}, -\frac{4}{9}\right)$
g) Range: $y \neq 0, y < -\frac{4}{9}, y \in R$
h) Differences between f(x) and g(x):
- 3 branches, # of asymptotes & restrictions, Local Maximums, Rate of Change

3. $y = -\frac{2}{(x-1)^2}$



a) Restrictions: $x \neq 1$ Domain: $x \in R, x \neq 1$
c) Behaviour of Asymptotes: As $x \to -1^-, h(x) \to -\infty$
As $x \to -1^+, h(x) \to \infty$
d) Rate of Change: x < -1 Negative Increasing Slope
x > -1 Positive Decreasing Slope
e) Range: y < 0
f) Differences from f(x):
- Asymptotes/Domain, Negative Function, Rate of Change
Differences from g(x)
- only one asymptote (root occurs 2x), no parabola part, slopes are different left to right, Change in slopes are similar

4. $y = \frac{2}{x^2+3x+2}$



i) Asymptotes: y = 0, x =-1, -2
ii)Domain/Range: $x \neq -1, -2, x \in R, y > 0, y < -8$
iii) Behaviour of Slope: x < -2 Positive Increasing
-2<x<-1.5 Positive Decreasing
-1.5 < x < -1 Negative Increasing
x > -1 Negative Decreasing
iv) End Behaviour As $x \to -\infty, k(x) \to 0$
As $x \to \infty, k(x) \to 0$
v) Intercepts: (0,1), No x-int