4.8.3 Trigonometric Proofs! (Continued)

$\dfrac{\sin^2 x}{1-\cos x}=1+\cos x$	$\tan x(\cot x+\tan x)=\sec^2 x$
$\dfrac{1-\cos^2 x}{1-\cos x}=1+\cos x$	$\tan x\left(\dfrac{1}{\tan x}+\tan x\right)=\sec^2 x$
$\dfrac{(1+\cos x)(1-\cos x)}{(1-\cos x)}=1+\cos x$	$1+\tan^2 x=\sec^2 x$
$1+\cos x=1+\cos x$	$1+\dfrac{\sin^2 x}{\cos^2 x}=\sec^2 x$
$LS=RS$	$\dfrac{\cos^2 x}{\cos^2 x}+\dfrac{\sin^2 x}{\cos^2 x}=\sec^2 x$
	$\dfrac{1}{\cos^2 x}=\sec^2 x$
	$\sec^2 x=\sec^2 x$
	$LS=RS$

4.8.3 Trigonometric Proofs! (Continued)

$(1+\sin x)(1-\sin x)=\dfrac{1}{\sec^2 x}$ $1-\sin^2 x=\dfrac{1}{\sec^2 x}$ $\cos^2 x=\dfrac{1}{\sec^2 x}$ $\dfrac{1}{\sec^2 x}=\dfrac{1}{\sec^2 x}$ $LS=RS$	$\sin^2 x(\csc^2 x+\sec^2 x)=\sec^2 x$ $\sin^2 x\csc^2 x+\sin^2 x\sec^2 x=\sec^2 x$ $\sin^2 x\left(\dfrac{1}{\sin^2 x}\right)+\sin^2 x\left(\dfrac{1}{\cos^2 x}\right)=\sec^2 x$ $1+\dfrac{\sin^2 x}{\cos^2 x}=\sec^2 x$ $\dfrac{\cos^2 x}{\cos^2 x}+\dfrac{\sin^2 x}{\cos^2 x}=\sec^2 x$ $\dfrac{1}{\cos^2 x}=\sec^2 x$ $\sec^2 x=\sec^2 x$ $LS=RS$
$\cos^2 x+\tan^2 x\cos^2 x=1$ $\cos^2 x+\left(\dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x=1$ $\cos^2 x+\sin^2 x=1$ $1=1$ $LS=RS$	$\tan^2 x\cos^2 x=1-\cos^2 x$ $\tan^2 x\left(1-\sin^2 x\right)=1-\cos^2 x$ $\dfrac{\sin^2 x}{\cos^2 x}(\cos^2 x)=1-\cos^2 x$ $\sin^2 x=1-\cos^2 x$ $1-\cos^2 x=1-\cos^2 x$ $LS=RS$

4.8.3 Trigonometric Proofs! (Continued)

$\csc x \sec x = \cot x + \tan x$ $\csc x \sec x = \left(\frac{\cos x}{\sin x}\right) + \left(\frac{\sin x}{\cos x}\right)$ $\csc x \sec x = \left(\frac{\cos^2 x}{\sin x \cos x}\right) + \left(\frac{\sin^2 x}{\sin x \cos x}\right)$ $\csc x \sec x = \frac{1}{\sin x \cos x}$ $\csc x \sec x = \left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right)$ $\csc x \sec x = \csc x \sec x$ $LS = RS$	$\tan x(\cot x + \tan x) = \sec^2 x$ $\tan x\left(\frac{1}{\tan x} + \tan x\right) = \sec^2 x$ $1 + \tan^2 x = \sec^2 x$ $1 + \frac{\sin^2 x}{\cos^2 x} = \sec^2 x$ $\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \sec^2 x$ $\frac{1}{\cos^2 x} = \sec^2 x$ $\sec^2 x = \sec^2 x$ $LS = RS$
$\tan x \sin x = \frac{1 - \cos^2 x}{\cos x}$ $\tan x \sin x = \frac{\sin^2 x}{\cos x}$ $\tan x \sin x = \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin x}{1}\right)$ $\tan x \sin x = \tan x \sin x$ $LS = RS$	$\sin x(\csc x - \sin x) = \cos^2 x$ $\sin x \csc x - \sin^2 x = \cos^2 x$ $\sin x\left(\frac{1}{\sin x}\right) - \sin^2 x = \cos^2 x$ $1 - \sin^2 x = \cos^2 x$ $\cos^2 x = \cos^2 x$ $LS = RS$

4.8.4 Proving Trigonometric Identities: Practice

Knowledge

1. Prove the following identities:
   (a) $\tan x \cos x = \sin x$ (b) $\cos x \sec x = 1$

   (c) $(\tan x)/(\sec x) = \sin x$

2. Prove the identity:
   (a) $\sin^2 x(\cot x + 1)^2 = \cos^2 x(\tan x + 1)^2$

   (b) $\sin2x – \tan2x = -\sin2x\tan2x$

   (c) $(\cos2x – 1)(\tan2x + 1) = -\tan2x$

   (d) $\cos^4 x – \sin^4 x = \cos^2 x – \sin^2 x$

3. Prove the identity
   (a) $\cos(x – y)/[\sin x \cos y] = \cot x + \tan y$

   (b) $\sin(x + y)/[\sin(x – y)] = [\tan x + \tan y]/[\tan x – \tan y]$

4. Prove the identity:
   (a) $\sec x / \csc x + \sin x / \cos x = 2 \tan x$

   (b) $[\sec x + \csc x]/[1 + \tan x] = \csc x$

   (c) $1/[\csc x – \sin x] = \sec x \tan x$

Application

5. Half of a trigonometric identity is given. Graph this half in a viewing window on $[-2\pi, 2\pi]$ and write a conjecture as to what the right side of the identity is. Then prove your conjecture.

   (a) $1 – (\sin^2 x / [1 + \cos x]) = ?$

   (b) $(\sin x + \cos x)(\sec x + \csc x) – \cot x – 2 = ?$

4.8.4 Proving Trigonometric Identities (Continued)

6. Prove the identity:
   (a) $[1 - \sin x] / \sec x = \cos 3x / [1 + \sin x]$

   (b) $-\tan x \tan y(\cot x - \cot y) = \tan x - \tan y$

7. Prove the identity:

   $\cos x \cot x / [\cot x - \cos x] = [\cot x + \cos x] / \cos x \cot x$

8. Prove the identity:

   $(\cos x - \sin y) / (\cos y - \sin x) = (\cos y + \sin x) / (\cos x + \sin y)$

Thinking

9. Prove the "double angle formulae" shown below:

$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\tan 2x = 2 \tan x / [1 - \tan^2 x]$$

Hint: $2x = x + x$